Chapter - 5
Number Play

Rules of Odd, Even Numbers:

Addition and Subtraction:

1. Even ± Even = Even (68+24=92)
2. Odd ± Odd = Even (35+87=122)
3. Even ± Odd = Odd (4+3=7, 9+54=63)

These rules are same for both Addition and Subtraction.

Multiplication:

1. Even x Even = Even (24x2=48)
2. Even x Odd = Even (10x5=50, 13x2=26)
3. Odd x Odd = Odd (33x77=2541)

Division:

1. Even ÷ Odd = Even (28/7 = 4)
2. Odd ÷ Odd = Odd (95/5 = 19)
3. Even ÷ Even = Even \ Odd (1004/2 = 502, 78/6 = 13)
4. Odd ÷ Even = Not a whole numbers

Note: the sign, '=>' written in the notes is to be read as, 'that implies'. Write this in your note as arrow sign drawn near this note. ⇒

Note: Use the three dots sign shown here, ∴ wherever needed in the place of Therefore.

Padlet

Pg. 115 Using our understanding of how parity behaves under different operations, identify which of the following algebraic expressions give an even number for any integer values for the letter-numbers.
(Identify which of the following algebraic expression give an even number)

i . 2a + 2b

Soln:

2a is always an even number (Since, Multiple of 2 is always a even number)
2b is always an even number (Since, Multiple of 2 is always a even number)
We know that, Even + Even = Even
Therefore, 2a + 2b = Even Number

ii . 3g + 5h

Soln:

3g - Odd Number	3g - Even Number	3g - Odd Number	3g - Even Number
5h - Odd Number	5h - Even Number	5h - Even Number	5h - Odd Number

We know that, Even + Even = Even and Odd + Odd = Even,
Odd + Even = Even + Odd = Odd.
Therefore, 3g + 5h = Even number if both 3g and 5h are Odd or both 3g and 5h are Even numbers. Else, 3g + 5h = Odd number.

Padlet

iii . 4m + 2n

Soln:

4m is always an even number (Since, Multiple of 4 is always a even number)

2n is always an even number (Since, Multiple of 2 is always a even number)

We know that, Even + Even = Even

Therefore, 4m + 2n = Even Number

iv . 2u - 4v

Soln:

2u is always an even number (Since, Multiple of 2 is always a even number)

4v is always an even number (Since, Multiple of 4 is always a even number)

We know that, Even - Even = Even

Therefore, 2u - 4v = Even Number

v . 13k - 5k

Soln:

13k - 5k = (13-5)k = 8k, which is always an even number (Since, Multiple of 8 is always a even number)

Therefore, 13k - 5k = Even Number

13k = Odd, 5k = Odd

Odd - Odd = Even

Padlet

vi . 6m - 3n

Soln:

6m is always an even number (Since, Multiple of 6 is always a even number)

3n will be Odd if n=Odd, 3n will be Even if n=Even

6m - Even Number	6m - Even Number
3n - Even Number	3n - Odd Number

We know that, Even - Even = Even

Therefore, 6m - 3n = Even Number if both 6m and 3n are Even.

Else, if 3n is Odd, 6m - 3n = Odd number.

vii . $x^2 + 2$

Soln:

2 is an Even number

x^2 will be Odd if x=Odd, x^2 will be Even if x=Even

x^2 - Even Number	x^2 - Odd Number
2 - Even Number	2 - Even Number

We know that, Even + Even = Even

Therefore, $x^2 + 2$ = Even Number if x^2 is Even.

Else, if x^2 is Odd, $x^2 + 2$ = Odd Number.

Padlet

ix . 4k x 3j

Soln:

4k is always an even number (Since, Multiple of 4 is always a even number)

3j will be Odd if j=Odd, 3j will be Even if j=Even

4k - Even Number	4k - Even Number
3j - Even Number	3j - Odd Number

We know that, Even x Even = Even, Even x Odd = Even

Therefore, 4k x 3j = Even Number. In Multiplication of two numbers, both the numbers have to be odd, to get the result as Odd Number.

viii . $b^2 + 1$

Soln:

1 is an Odd number

b^2 will be Odd if b=Odd, b^2 will be Even if b=Even

b^2 - Even Number	b^2 - Odd Number
1 - Odd Number	1 - Odd Number

We know that, Odd + Odd = Even

Therefore, $b^2 + 1$ = Even Number if b^2 is Odd.

Else, if b^2 is Even, $b^2 + 1$ = Odd Number.

Padlet

Pg. 122 Figure it out:

1. The sum of four consecutive numbers is 34. What are these numbers?

Soln:

Let the four consecutive numbers be x, x+1, x+2, x+3

Given: $(x) + (x+1) + (x+2) + (x+3) = 34$

$x + x+1 + x+2 + x+3 = 34$

$4x+6 = 34$

$4x = 34 - 6$

$4x = 28$

$x = 28 \div 4$

$x = 7$

We found x = 7,

So, $x+1 = 7+1 = 8$,

$x+2 = 7+2 = 9$,

$x+3 = 7+3 = 10$

Therefore, the four consecutive numbers which gives the sum of them as 34 are 7,8,9,10

Padlet

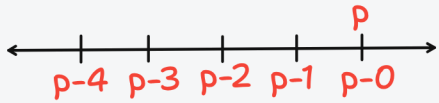
Pg. 122 Figure it out:

2. Suppose p is the greatest of five consecutive numbers. Describe the other four numbers in terms of p.

Soln:

p is the greatest of five consecutive numbers.

Then, the other four numbers are p-4, p-3, p-2, p-1.



3. For each statement below, determine whether it is always true, sometimes true, or never true. Explain your answer. Mention examples and non-examples as appropriate. Justify your claim using algebra.

(i) The sum of two even numbers is a multiple of 3.

Ans: Sometimes true

Example: 2 + 4 = 6 (multiple of 3)

6 + 4 = 10 (not a multiple of 3)

(ii) If a number is not divisible by 18, then it is also not divisible by 9.

Ans: Sometimes true

Example: 35 (not divisible by 18, so not divisible by 9)

45 (not divisible by 18, but divisible by 9)

(iii) If two numbers are not divisible by 6, then their sum is not divisible by 6.

Ans: Sometimes true

Example: 7 + 9 = 16 (not divisible by 6)

5 + 1 = 6 (divisible by 6)

(iv) The sum of a multiple of 6 and a multiple of 9 is a multiple of 3.

Ans: Always true

Example: 6a + 9b = 3(2a + 3b) which is divisible by 3

Therefore, sum of a multiple of 6 and a multiple of 9 is a multiple of 3.)

(v) The sum of a multiple of 6 and a multiple of 3 is a multiple of 9.

Ans: Sometimes true

Example: 6 + 3 = 9 (multiple of 9)

12 + 3 = 15 (not a multiple of 9)

Padlet

4. Find a few numbers that leave a remainder of 2 when divided by 3 and a remainder of 2 when divided by 4. Write an algebraic expression to describe all such numbers.

Soln:

Number that leaves a remainder of 2 when divided by 3 is, 3a+2 -> i

Number that leaves a remainder of 2 when divided by 4 is, 4a+2 -> ii

i and ii = (Multiple of both 3 and 4) + 2

= (LCM of 3,4) + 2

Expression = 12c + 2

If c=1, 12(1)+2 = 12+2 = 14

If c=2, 12(2)+2 = 24+2 = 26

If c=15, 12(15)+2 = 180+2 = 182

If c=20, 12(20)+2 = 240+2 = 242

14, 26, 182, 242 are some of the numbers that leaves a remainder of 2 when divided by 3 and a remainder of 2 when divided by 4.

In 12c+2, you may put any values in c to get such a number

Padlet

5. "I hold some pebbles, not too many, When I group them in 3's, one stays with me. Try pairing them up — it simply won't do, A stubborn odd pebble remains in my view. Group them by 5, yet one's still around, But grouping by seven, perfection is found. More than one hundred would be far too bold, Can you tell me the number of pebbles I hold?"

Soln:

Let x be the number

x leaves a remainder of 1 when divided by 2,3 and 5, and

x is divisible by 7, and

x is lesser than or equal to 100 (i.e., $x \leq 100$)

LCM of 2,3,5 = 30

$x = 30k+1$

For k=1, 30(1)+1 = 31 (not divisible by 7)

For k=2, 30(2)+1 = 61 (not divisible by 7)

For k=3, 30(3)+1 = 91 (divisible by 7)

91 is less than 100, which is divisible by 7, and leaves a remainder 1 when divided by 2,3 and 5.

91 is the number of pebbles that it holds.

Padlet

6. Tathagat has written several numbers that leave a remainder of 2 when divided by 6. He claims, "If you add any three such numbers, the sum will always be a multiple of 6." Is Tathagat's claim true?

Soln:

A number that leaves a remainder 2 when divided by 6 = 6k+2

Let three such a numbers be 6a+2, 6b+2, 6c+2

= (6a+2) + (6b+2) + (6c+2)

= 6a+2+6b+2+6c+2

=6a+6b+6c+6

=6(a+b+c)+6

=6((a+b+c)+1) (Multiple of 6)

Therefore, Tathagat claim, adding any three such numbers, the sum will always be a multiple of 6 is true.

7. When divided by 7, the number 661 leaves a remainder of 3, and 4779 leaves a remainder of 5. Without calculating, can you say what remainders the following expressions will leave when divided by 7? Show the solution both algebraically and visually. (i) 4779 + 661 (ii) 4779 – 661

Soln:

i) 4779+661

When divided by 7, 4779 gives the remainder of 5, $4779 = 7a+5$ -> 😄

When divided by 7, 661 gives the remainder of 3, $661 = 7b+3$ -> 😳

Adding 😄 & 😳

$4779 + 661 = 7a + 5 + 7b + 3$

$= 7a + 7b + 8$

$= 7a + 7b + 7 + 1$

$= 7(a+b+1) + 1$

Therefore, the remainder is 1

ii) 4779-661

$4779 - 661 = (7a + 5) - (7b + 3)$

$= 7a + 5 - 7b - 3 \quad = 7a - 7b - 2$

$= 7(a-b) + 2$

Therefore, the remainder is 2

Padlet

8. Find a number that leaves a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, and a remainder of 4 when divided by 5. What is the smallest such number? Can you give a simple explanation of why it is the smallest?

Soln:

The expression of a number that leaves a remainder of 2 when divided by 3 is,

$3k+2$ (1 less than multiple of 3)

The expression of a number that leaves a remainder of 3 when divided by 4 is,

$4k+3$ (1 less than multiple of 4)

The expression of a number that leaves a remainder of 4 when divided by 5 is,

$5k+4$ (1 less than multiple of 5)

Smallest number = LCM of (3,4,5) - 1

$= 60 - 1$

$= 59$

Therefore, 59 is the smallest number satisfying the given conditions.

Divisibility of 9

A number is divisible by 9 if and only if the sum of its digits is divisible by 9.

'iff' stands for 'if and only if'

Pg. 126 Figure it out:

1. Find, without dividing, whether the following numbers are divisible by 9.

(i) 123

Soln:

Sum of the digits = 1+2+3

= 6

6 is not divisible by 9

Therefore, 123 is not divisible by9

(ii) 405

Soln:

Sum of the digits = 4+0+5

= 9

9 is divisible by 9

Therefore, 405 is divisible by9

(iii) 8888

Soln:

Sum of the digits = 8+8+8+8 = 32

3+2=5 is not divisible by 9

32 is not divisible by 9

Therefore, 8888 is not divisible by9

Padlet

(iv) 93547

Soln:

Sum of the digits = 9+3+5+4+7

= 28

2+8=10 is not divisible by 9

28 is not divisible by 9

Therefore, 93547 is not divisible by9

(v) 358095

Soln:

Sum of the digits = 3+5+8+0+9+5

= 30

3+0=3 is not divisible by 9

30 is not divisible by 9

Therefore, 358095 is not divisible by9

4. How many multiples of 9 are there between the numbers 4300 and 4400?

Soln:

4300 to 4400

Smallest number divisible by 9 = 4302

Largest number divisible by 9 = 4392

Number of multiples of 9 $= \frac{4392 - 4302}{9} + 1$

$= \frac{90}{9} + 1$

$= 10 + 1$

$= 11$

Therefore, there are 11 multiples of 9 between the numbers 4300 and 4400.

Digital Roots

Pg. 130 2. Write the digital roots of any 12 consecutive numbers. What do you observe? We saw that the digital root of multiples of 9 is always 9.

10	11	12	13	14	15	16	17	18	19	20	21
1	2	3	4	5	6	7	8	9	1	2	3

Therefore, it is repeating pattern of consecutive numbers (i.e., 1,2,3,4,5,6,7,8,9)

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Digital Roots

Pg. 130 4. What are the digital roots of numbers that are 1 more than a multiple of 6? What do you notice? Try to explain the patterns noticed.

Numbers 1 more than multiple of 6	7	13	19	25	31	37
Digital root	7	4	1	7	4	1

Therefore, it is a repeating pattern of 7, 4, 1

Digital Roots
Take a number. Add its digits repeatedly till you get a single-digit number. This single-digit number is called the digital root of the number. For example, the digital root of the number 489710 will be 2 (4 + 8 + 9 + 7 + 1 + 0 = 29, 2 + 9 = 11, 1 + 1 = 2)

Figure it out

Pg. 131 2. Write any number. Generate a sequence of numbers by repeatedly adding 11. What would be the digital roots of this sequence of numbers? Share your observations.

The digital roots forms the repeating pattern of 7, 9, 2, 4, 6, 8, 1, 3, 5.
The digital roots generated by the sequence of numbers by repeatedly adding 11 is given in table below.

Padlet

Number	Digital root
7	7
18	9
29	2 (11=1+1)
40	4
51	6
62	8
73	1 (10=1+0)
84	3 (12=1+2)
95	5 (14=1+4)
106	7
117	9
128	2 (11=1+1)

Figure it out

Pg. 131 3. What will be the digital root of the number 9a + 36b + 13?

Soln:

$= 9a + 36b + 9 + 4$

$= 9(a+4b+1) + 4$

$= 9 + 4$

$= 13$

$13 = 9 + 4$

Therefore, the digital root of $9a + 36b + 13 = 4$

Figure it out

Pg. 131 4. Make conjectures by examining if there are any patterns or relations between (i) the parity of a number and its digital root.

Soln:

Number	Digital root
Even: 46	4 + 6 = 10 = 10 = 1 (odd)
Even: 26	2 + 6 = 8 (even)
Odd: 47	4 + 7 = 11 = 2 (even)
Odd: 27	2 + 7 = 9 (odd)

Hence, there is no relation between the parity of a number and its digital roots.

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Figure it out

Pg. 131 4. Make conjectures by examining if there are any patterns or relations between (ii) the digital root of a number and the remainder obtained when the number is divided by 3 or 9.

Soln:

The digital root gives the remainder when the number is divided by 9.
(except when digital root = 9)
=> Digital root = 9, then it is divisible by 9
=> the remainder = 0
=> Digital root = 3, then it is divisible by 3
=> the remainder = 0
Digital root of 43 = 4 + 3 = 7
43 ÷ 9 : Remainder = 7
Digital root of 45 = 4 + 5 = 9
45 ÷ 9 : Remainder = 0

Padlet